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The spin-quadrupole interaction and the 0^+ excitations in ^{158}Gd

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Abstract. In this work, the effect of spin-quadrupole forces on the 0^+ sates in ^{158}Gd has been investigated. For this purpose, the model Hamiltonian including monopole pairing, quadrupole-quadrupole and spin-quadrupole forces has been diagonalized in one phonon basis. In conclusion, for the distribution of energies of the states and their collective properties, fairly good results have been obtained.

PACS. 21.60.Jz Hartree-Fock and random-phase approximations – 21.60.Ev Collective models

1 Introduction

Collective states in deformed and transitional nuclei have been extensively studied in numerous experimental and theoretical works. Although there were many experimental data on lower 0^+ states [1,2], understanding of the mechanism of the 0^+ states in even-even deformed nuclei is still a challenge for theory. In other words, the nature of the 0^+ states is the one of the most controversial subject in these nuclei. In particular, the structure of the lowest one has become a research field by itself [3]. A summary of recent discussions on the nature of the 0^+ states and the status of the problem are presented in [3]. It is interesting to note that, while the nature of the lowest 0^+ state has not been understood certainly yet, the observation of a large number of 0^+ states in a recent (p,t) experiment [4] in ^{158}Gd made the explanation of their mechanism difficult and stimulated new studies on this field. In addition, the most recent (p,t) experiment conducted by Wirth *et al.* also reveals about ten 0^+ states below 2.5 MeV for some actinide nuclei [5]. According to these results, it is natural to expect that similar numbers of 0^+ excitations may appear throughout deformed regions. For that reason, it is important to understand the origin of such a large number of 0^+ modes. So far, although many theoretical studies on the 0^+ excitations in deformed nuclei exist in the last decades, there is not any study considering the situation up to ~ 3 MeV because of the unavailability of data in that range [1–3]. Recently the 0^+ states in ^{158}Gd have been studied by Zamfir *et al.* [6] using the Geometric Collective Model (GCM), sd-IBM and spdf-IBM. According to the results of this study, GCM and sd-IBM each can treat only 6 states up to 3.1 MeV. Moreover, spdf-IBM could give 9 or 10 states up to the energy range in question. Because of this result Zamfir *et al.* concluded that many of the 0^+ states might have a two-phonon octupole character. Nevertheless, this conclusion is a weak argument because their models do not include the two-quasiparticle (2qp) components. Another theoretical work on this subject belongs to Sun *et al.* [7]. In this work, the 0^+ excitations in ^{158}Gd have been investigated in terms of 2 and 4qp excitations using the Projected Shell Model in the framework of the Tamm-Dancoff approximation (TDA). A further theoretical study has been made in [8]. In that work Iudice *et al.* used QPM including monopole and quadrupole pairing with a quadrupole-quadrupole force term. Making a detailed analysis, they presented the calculations on the microscopic properties including energies, $E2$, $E0$ transitions and two-nucleon spectroscopic factors with the shell and multiphonon structure of the 0^+ states. The last study has been made in [9] by using the pairing-plus-quadrupole model (PPQ), including only monopole pairing; a good description has been given for the distribution and the nature of the 0^+ states.

As a result, the whole developments mentioned above imply that new microscopic models and interactions that can give new contributions are necessary. In particular, the interactions that can generate new states below ~ 2 MeV for a consistent explanation of the distribution of the states and the collective properties of the lowest states in question are needed. In fact, as stated above, the experiment which were conducted by Lesher *et al.* showed that ^{158}Gd has seven 0^+ states below ~ 2 MeV. This fact lead us to explore new interactions to increase the number of states below 2 MeV. To this purpose, one of the useful interaction might be the spin-quadrupole (SQI) interaction. This interaction was first studied by Kisslinger in single-closed-shell spherical nuclei, many years ago [10]. The effects of the spin-quadrupole interaction in deformed nuclei

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have been searched by Pyatov and collaborators [11–18]. However, this interaction has been neglected by many researchers so far. Using the spin-quadrupole interaction, Pyatov *et al.* pointed out that a significant decrease of the diagonal amplitude can be obtained. Moreover, this interaction can generate a new 0^+ state below the energy gap in even-even nuclei. The latter property of this interaction is very important for the investigation on the 0^+ states in ^{158}Gd.

In this paper, in order to study the nature of the 0^+ excitations in the ^{158}Gd isotope and investigate the general behavior of the quantities that characterize them, we use the model Hamiltonian including monopole pairing, quadrupole-quadrupole and spin-quadrupole forces in the framework of the Random Phase Approximation (RPA). In this direction, the energies, the quadrupole $B(E2; K^\pi = 0^+ \to 2^+_{gsb})$ and the monopole $\rho^2(E0)$ transitions to the ground-state band and the spectroscopic factors of (t, p) and (p, t) reactions were calculated.

2 Model

The model Hamiltonian in the second quantization representation can be given as

$$H = H_{sp} + H_{pair} + H_{QQ} + H_{SQ}. \tag{1}$$

Here

$$H_{sp} = \sum_{s\sigma} (E(s) - \lambda_N) a^\dagger_{s\sigma} a_{s\sigma} \,, \tag{2}$$

$$H_{pair} = -G_N \sum_{s} a^\dagger_{s+} a^\dagger_{s-} a_{s-} a_{s+} \,, \tag{3}$$

$$H_{QQ} = -\frac{1}{2} \sum_{\lambda} \sum_{\mu} \chi^\lambda Q^\dagger_{\lambda\mu} Q_{\lambda\mu} \,, \tag{4}$$

$$H_{SQ} = -\frac{1}{2} \chi^2_\sigma T^\dagger_\sigma(\lambda\mu 2) T_\sigma(\lambda\mu 2) \,, \tag{5}$$

and the multipole moment and the spin-multipole interaction operator, respectively, are

$$Q_{\lambda\mu} = \sum_{\substack{ss' \\ \sigma\sigma'}} \langle s\sigma | f^{\lambda\mu} | s'\sigma \rangle a^\dagger_{s\sigma} a_{s'\sigma'} \,, \tag{6}$$

$$T_\sigma(\lambda\mu 2) = \sum_{s,s',\sigma} \left\{ t^{\lambda\mu}(s,s') a^\dagger_{s\sigma} a_{s'\sigma} + \sigma \bar{t}^{\lambda\mu}(s,s') a^\dagger_{s\sigma} a_{s'-\sigma} \right\}. \tag{7}$$

The labels $s\sigma$ stand for the single-particle asymptotic quantum numbers $s = Nn_z\Lambda\uparrow$ for $K^\pi = \Lambda + \frac{1}{2}$ and $s = Nn_z\Lambda\downarrow$ for $K^\pi = \Lambda - \frac{1}{2}$ with $\sigma = \pm 1$. Naturally, the indices of the operator (6) and (7) are $\lambda = 2$ and $\mu = 0$ for the quadrupole-quadrupole and also for spin-quadrupole forces. In addition, in (6) the selection rule for the quadrupole-quadrupole matrix elements $f(s,s') \equiv \langle s\sigma | r^2 Y_{20} | s'\sigma \rangle = \langle s\sigma | f^{20} | s'\sigma \rangle$ is $K_2 \pm \mu = K_1$. Spin-multipole matrix elements in (7) are calculated according to

$$\begin{aligned}
t^{2\mu}(s,s') &\equiv \left\langle s+ \left| r^2 \left[\{\sigma Y_2\}_{2\mu} + (-1)^\mu \{\sigma Y_2\}_{2-\mu} \right] \right| s'+ \right\rangle \\
&= \left\langle s- \left| r^2 \left[\{\sigma Y_2\}_{2\mu} + (-1)^\mu \{\sigma Y_2\}_{2-\mu} \right] \right| s'- \right\rangle , \\
\bar{t}^{\,2\mu}(s,s') &\equiv \left\langle s+ \left| r^2 \left[\{\sigma Y_2\}_{2\mu} + (-1)^\mu \{\sigma Y_2\}_{2-\mu} \right] \right| s'- \right\rangle \\
&= -\left\langle s- \left| r^2 \left[\{\sigma Y_2\}_{2\mu} + (-1)^\mu \{\sigma Y_2\}_{2-\mu} \right] \right| s'+ \right\rangle , \\
t^{2\mu}(s,s') &= -t^{2\mu}(s',s) \,, \qquad \bar{t}^{\,2\mu}(s,s') = \bar{t}^{\,2\mu}(s',s) \,.
\end{aligned}$$

Here for the spin-quadrupole interaction that can be related with the $K^\pi = 0^+$ ($\mu = 0$) states, the part of the operator takes the form,

$$\{\sigma Y_2\}_{20} = \frac{1}{\sqrt{2}} (\sigma_+ Y_{2,-1} - \sigma_- Y_{2,-1}) \,.$$

Now, if the Hamiltonian (1) is written for neutrons and also protons, the total Hamiltonian of the model can be given as

$$H = H_{sp}(n) + H_{sp}(p) + H_{pair}(n) + H_{pair}(p) + H_{QQ} + H_{SQ} \,. \tag{8}$$

In order to obtain the energies and wave functions of the excited 0^+ sates, we follow the RPA method in this work. In the RPA method, the first step consists in expressing the Hamiltonian in terms of the quasiparticle operators $\alpha_{s\sigma}$ and $\alpha^\dagger_{s\sigma}$, by making use of the Bogolyubov canonical transformation

$$a_{s\sigma} = u_s \alpha_{s\sigma} + \sigma v_s \alpha^\dagger_{s-\sigma} \,.$$

In deriving the quasiparticle energies and the u , v that are the BCS particle occupation probabilities, we take into account monopole ($\lambda = \mu = 0$) pairing.

The second step consists in constructing the RPA phonon operators

$$Q^\dagger_n = \frac{1}{2} \sum_{ss'} \left(\psi^n_{ss'} A^\dagger(ss') - \varphi^n_{ss'} A(ss') \right) \,,$$

where n labels the RPA roots, *i.e.* it denotes the n-th 0^+ state. $A^\dagger(ss')(A(ss'))$ is the quasiparticle pair of the creation (annhilation) operator. In the result, the

Hamiltonian (8) in the phonon representation becomes,

$$
\begin{aligned}
H = & \sum_q \varepsilon(q) B(q,q) \\
& -\frac{1}{4}\sum_{nn'}\Big\{ G_N \sum_{ss'}\left[(u_s^2 - v_s^2)(u_{s'}^2 - v_{s'}^2) g_{ss}^n g_{s's'}^n\right] \\
& + G_N \sum_{ss'} w_{ss}^n w_{s's'}^n + G_Z \sum_{rr'}\Big[(u_r^2 - v_r^2)(u_{r'}^2 - v_{r'}^2) g_{rr}^n g_{r'r'}^{n'}\Big] \\
& + G_Z \sum_{rr'} w_{rr}^n w_{r'r'}^{n'} \Big\} Q_n^\dagger Q_{n'} \\
& -\frac{\chi}{4}\sum_{nn'}\sum_{qq'q_2q_2'} u_{qq'} u_{q_2q_2'} f(q,q') g_{qq'}^n f(q_2,q_2') g_{q_2q_2'}^{n'} \\
& \times \Big\{ Q_n^\dagger Q_{n'} + Q_{n'}^\dagger Q_n \Big\} \\
& -\frac{\chi_\sigma}{4}\sum_{nn'}\sum_{qq'q_2q_2'} t^{20}(q,q') u_{qq'}^{(-)} w_{qq'}^n \\
& \times \sum_{q_2q_2'} t^{20}(q_2,q_2') u_{q_2q_2'}^{(-)} w_{q_2q_2'}^n \Big\{ Q_n^\dagger Q_{n'} + Q_{n'}^\dagger Q_n \Big\} . \qquad (9)
\end{aligned}
$$

In the Hamiltonians (3) and (7), G_N (G_Z) denotes the pairing strength constant for neutrons (protons) which is

$$\frac{1}{G_N} = \sum_s \frac{1}{2\varepsilon_s} . \qquad (10)$$

In addition, χ and χ_σ denote the quadrupole-quadrupole and spin-quadrupole interactions strength constants, respectively. In the equations, the index q means that the expression is valid for neutrons and also protons. Moreover, the functions in (9) are given as

$$
\begin{aligned}
g_{ss'}^n &= \psi_{ss'}^n + \varphi_{ss'}^n , \qquad & w_{ss'}^n &= \psi_{ss'}^n - \varphi_{ss'}^n , \\
u_{qq'} &= u_q v_{q'} + u_{q'} v_q , \qquad & u_{qq'}^{(-)} &= u_q v_{q'} - u_{q'} v_q .
\end{aligned}
$$

Here, u_s and v_s are the famous BCS particle occupation probabilities given by

$$
\begin{aligned}
u_s^2 &= \frac{1}{2}\left\{1 + \frac{(E(s) - \lambda_N)}{\varepsilon_s}\right\}, \\
v_s^2 &= \frac{1}{2}\left\{1 - \frac{(E(s) - \lambda_N)}{\varepsilon_s}\right\},
\end{aligned}
$$

where $E(s)$ and λ_N denote the single-particle energies and the chemical potential, respectively. On the other hand, the quasiparticle energies $\varepsilon(s)$ are equal to

$$\varepsilon(s) = \sqrt{(E(s) - \lambda_N)^2 + \Delta_N^2}.$$

In the RPA method the collective 0^+ states are considered as one-phonon excitations and the wave function of the n-th 0^+ state is described by

$$|\psi_n\rangle = Q_n^\dagger |\psi_0\rangle = \frac{1}{\sqrt{2}} \sum_{ss',\tau} [\psi_{ss'}^n(\tau) A_{ss'}^\dagger - \varphi_{ss'}^n(\tau) A_{ss'}] |\psi_0\rangle , \qquad (11)$$

where $Q_n^\dagger$ is the phonon creation operator, and $|\psi_0\rangle$ is the phonon vacuum, which corresponds to the ground state of an even-even nucleus. The isotopic index τ takes the values n (p) for neutrons (protons). The two-quasiparticle amplitudes $\psi_{ss'}^n$ and $\varphi_{ss'}^n$ are normalized by

$$\sum_{ss'\tau} \left(\psi_{ss'}^n(\tau)\right)^2 - \varphi_{ss'}^n(\tau)^2) = 1 . \qquad (12)$$

Solving the equation of motion for the Hamiltonian (9)

$$\left[H, Q_n^\dagger\right] = \omega Q_n^\dagger , \qquad (13)$$

the secular equation that gives the excitation energies is found as

$$(1 - \chi F(\omega))(1 - \chi_\sigma S(\omega)) = \chi\chi_\sigma W^2(\omega) , \qquad (14)$$

where

$$S(\omega) = \sum_{qq'} \frac{\left[t^{20}(q,q')\right]^2 (u_{qq'}^{(-)})^2 2\varepsilon_{qq'}}{\varepsilon_{qq'}^2 - \omega^2} , \qquad (15)$$

$$W(\omega) = \sum_{qq'} \frac{2\omega f(q,q') t^{20}(q,q') u_{qq'} u_{qq'}^{(-)}}{\varepsilon_{qq'}^2 - \omega^2} . \qquad (16)$$

The structure of the $F(\omega)$ term in (14) and other details can be found in [2].

3 Collective properties of the 0^+ excitations

In order to search for the collectivity and to investigate the nature of 0^+ states, we calculated the electromagnetic $B(E2)$, $\rho^2(E0)$ transition probabilities and the (t,p) and (p,t) reaction strengths relative to the ground-to-ground strengths; $\tilde{S}_n(t,p) = S_n(t,p)/S_{g.s}(t,p)$ and $\tilde{S}_n(p,t) = S_n(p,t)/S_{g.s}(p,t)$. Consequently, there can exist a few situations for the nature of the 0^+ states in this model:

a) If a state has a pure β-vibrational nature, the two-nucleon transfer strengths should be relatively small and $B(E2)$ and $\rho^2(E0)$ transitions should be relatively large:

$$
\begin{aligned}
& B(E2; 0^+0 \to 2^+0) = \\
& \frac{e^2}{2}\left|(1+e_p)\sum_{rr'} g_{rr'}^n f(r,r') u_{rr'} + e_n \sum_{ss'} g_{ss'}^n f(s,s') u_{ss'}\right|^2, \\
& B(E0) = \rho^2(E0) = \\
& \frac{e^2}{2R_0^4}\left|(1+e_p)\sum_{rr'} \langle r|r^2|r'\rangle g_{rr'}^n u_{rr'} + e_n \sum_{ss'} \langle s|r^2|s'\rangle g_{ss'}^n u_{ss'}\right|^2,
\end{aligned}
$$

where, e_p and e_n are the proton and the neutron effective charges, respectively. In the calculations we take their values as 0.1. In the $\rho^2(E0)$ transition, R_0 is the nucleus radius mean value. In addition, the index r (s) denotes proton (neutron) states.

b) If a state has a pure pairing vibrational nature, the $B(E2)$ and $\rho^2(E0)$ transitions should be relatively small

and at least one or both of the two-nucleon transition strengths should be relatively large:

$$\tilde{S}_n(t,p) = \frac{S_{gs\to es}(t,p)}{S_{gs\to gs}(t,p)} = \left| \frac{\sum_s \left(u_s^2 \psi_{ss}^n - v_s^2 \varphi_{ss}^n\right)}{\frac{\Delta_N}{G_N}} \right|^2 ,$$

$$\tilde{S}_n(p,t) = \frac{S_{gs\to es}(p,t)}{S_{gs\to gs}(p,t)} = \left| \frac{\sum_s \left(u_s^2 \varphi_{ss}^n - v_s^2 \psi_{ss}^n\right)}{\frac{\Delta_N}{G_N}} \right|^2 .$$

Thus, the states whose $\tilde{S}_n(t,p)$ and $\tilde{S}_n(p,t)$ values are relatively large ($\geqslant$ 1–2%) were accepted as pairing vibrational type. In fact it is difficult to find a pure pairing vibrational or a pure β-vibrational state, because collective 0^+ states frequently have a mixed structure in the RPA due to the coupling between pairing and β-vibrational modes.

c) In the framework of the present model another possibility can be the spin-quadrupole vibrational mode. Its properties will be discussed in the next section.

d) Naturally if any state has relatively weak $B(E2)$ and $\rho^2(E0)$ values and also relatively weak two-nucleon transfer strengths, it should be a 2qp structure. In this context, it is beneficial to state that many of the 0^+ states have 2qp structure in RPA.

4 Calculations and discussion

In order to study the excited 0^+ states in ^{158}Gd, we used a model Hamiltonian including monopole pairing, quadrupole-quadrupole and spin-quadrupole forces. The energies and wave functions of the 0^+ states were calculated in RPA with the wave functions consisting of only one-phonon terms. To obtain the numerical results, the single-particle Hamiltonian developed by Boisson and Piepenbring was used [19]. To simplify the matrix elements, the asymptotic basis of eigenvectors was preferred. All of the calculations have been performed using the deformation value $\varepsilon_2 = 0.27$. In addition, all states of the $N = 4$, 5 and 6 shells for neutrons and protons (64 levels for each) were taken into account. For the calculation of the monopole pairing strength, gap parameters are used as $\sim 11.2/A^{1/2}$ MeV for protons and $\sim 12/A^{1/2}$ MeV for neutrons, respectively. The experimental data have been taken from [3,4,8,20,21]. To investigate the effect of the spin-quadrupole interaction on the 0^+ states, the calculations were performed in two ways. In the first one, the energies and related transition strengths have been calculated using the pairing-plus-quadrupole model (PPQ) *i.e.*, without spin-quadrupole forces. In the second one, the calculations in question have been made using the model including monopole pairing, quadrupole-quadrupole and also spin-quadrupole forces (PPQSQ). For comparison, the results of the two models have been presented in table 1. In addition, to search for the dependence of the properties of 0^+ states on the strengths of the quadrupole-quadrupole and spin-quadrupole forces the following calculations have been made. First, while the spin-quadrupole strength constant is changed, the quadrupole-quadrupole strength constant is kept fixed. These values are given in tables 1 and 2. Second, the inverse procedure has been followed, *i.e.* the spin-quadrupole strength constant is kept fixed. These values are presented in table 2. In tables 1 and 2, the corresponding theoretical and experimental values of the energies in the first column, the $B(E2; K^\pi = 0^+ \to 2^+_{gsb})$ transitions in the second column, the monopole transitions $\rho^2(E0)$ in the third column, the (t,p) spectroscopic factors in the fourth column and the (p,t) spectroscopic factors in the fifth column are given. The calculations have been performed for fifteen states because the observed top experimental state is at $\sim$ 3.1 MeV. The (t,p) and (p,t) reactions spectroscopic factors were given relative to the ground-to-ground strengths $\tilde{S}_n(t,p) = S_n(t,p)/S_{g.s}(t,p)$ and $\tilde{S}_n(p,t) = S_n(p,t)/S_{g.s}(p,t)$.

It is clear from table 1 that according to the PPQ model there is only one 0^+state which lies below the gap energy level (2Δ) and it is generated essentially by quadrupole-quadrupole interaction [1,2]. The first values on the left side in table 1 belong to the PPQ model. Comparison reveals that the second state of the present model has not any corresponding state in the PPQ model, *i.e.* the state in question is produced by the spin-quadrupole interaction. This property was also reported in [14,15]. Thus, as is seen from the tables there are two calculated 0^+ states below the gap and each one has relatively strong $B(E2)$ and $\rho^2(E0)$ transition probabilities and also large (t,p) and (p,t) strengths. Analysing the tables it is clear that while the spin-quadrupole and quadrupole-quadrupole strengths are changing the most sensitive states are the two lowest ones. Especially they depend sensitively on the spin-quadrupole interaction strength. Moreover, it is clear from the tables that these states behave in the same way while the strength parameters are changing. This situation reveals also these states are not pure because of the coupling. As is seen from eq. (14), the interference term in (16) causes this coupling in which there exist both quadrupole-quadrupole and spin-quadrupole matrix elements. Thus the strength of the spin-quadrupole interaction affects the energies and its collectivity of the second and also the first state. Variation of the strength of the quadrupole-quadrupole interaction results in a similar influence. Specifically, it is possible to conclude from the calculations for the collective properties of the spin-quadrupole vibrational mode that it has relatively large $B(E2)$ strength, considerable spectroscopic factors, and weak $B(E0)$ strength. In addition, the quadrupole collectivity and the values of spectroscopic factors are weaker than those of the first one.

In general, the energies of the first two states have a tendency to decrease as the strength of the forces increases. In addition, the energies of a few states are also changing although they are not considerable. If the calculated values of the PPQSQ model in table 1 are accepted as a base, the best agreement between the theory and data is obtained for the $\tilde{S}_n(p,t)$ values of the, 4., 6., 7., 8., and 11., states [4]. Furthermore, the theoretical energies

Table 1. Calculated values of the excited 0^+ states of ^{158}Gd.

PPQ $\chi = 3.1A^{-4/3}\hbar\omega_0$					PPQSQ $\chi = 2.5A^{-4/3}\hbar\omega_0$, $\chi_\sigma = 1.54A^{-4/3}\hbar\omega_0$				
Energy (MeV)	$B(E2)$ (w.u.)	$\rho^2(E0)$	$\tilde{S}(t,p)$	$\tilde{S}(p,t)$	Energy (MeV)	$B(E2)$ (w.u.)	$\rho^2(E0)$	$\tilde{S}(t,p)$	$\tilde{S}(p,t)$
1.19	2.1235	0.0360	0.2621	0.2361	1.37	1.1637	0.0197	0.1477	0.1433
1.79	0.0123	0.0040	0.0027	0.0048	1.71	0.2463	0.0028	0.0302	0.0427
1.93	0.0550	0.0001	0.0007	0.0009	1.80	0.0028	0.0124	0.0003	0.0006
1.96	0.0020	0.0000	0.0000	0.0030	1.93	0.0575	0.0002	0.0007	0.0009
1.97	0.0478	0.0024	0.0058	0.0058	1.96	0.0010	0.0000	0.0003	0.0032
2.21	0.0303	0.0002	0.0000	0.0000	1.97	0.0453	0.0026	0.0023	0.0032
2.36	0.0902	0.0004	0.0095	0.0349	2.21	0.0289	0.0002	0.0000	0.0000
2.71	0.0035	0.0000	0.0094	0.0146	2.41	0.0453	0.0002	0.0028	0.0381
2.76	0.0089	0.0000	0.0000	0.0057	2.71	0.0036	0.0000	0.0095	0.0145
3.00	0.0683	0.0005	0.0151	0.0044	2.81	0.0627	0.0000	0.0076	0.0007
3.06	0.0077	0.0006	0.0020	0.0001	3.02	0.0373	0.0007	0.0059	0.0013
3.13	0.0005	0.0000	0.0007	0.0117	3.09	0.0030	0.0009	0.0041	0.0001
3.16	0.0084	0.0030	0.0001	0.0008	3.13	0.0004	0.0000	0.0006	0.0117
3.21	0.0000	0.0000	0.0084	0.0030	3.16	0.0086	0.0031	0.0000	0.0004
3.28	0.0002	0.0000	0.0009	0.0003	3.21	0.0000	0.0000	0.0083	0.0027
				$\sum_n S_n = 0.33$					$\sum_n S_n = 0.26$

Table 2. Calculated values of the excited 0^+ states of ^{158}Gd.

PPQSQ $\chi = 2.5A^{-4/3}\hbar\omega_0$, $\chi_\sigma = 1.56A^{-4/3}\hbar\omega_0$					PPQSQ $\chi = 2.8A^{-4/3}\hbar\omega_0$, $\chi_\sigma = 1.56A^{-4/3}\hbar\omega_0$				
Energy (MeV)	$B(E2)$ (w.u.)	$\rho^2(E0)$	$\tilde{S}(t,p)$	$\tilde{S}(p,t)$	Energy (MeV)	$B(E2)$ (w.u.)	$\rho^2(E0)$	$\tilde{S}(t,p)$	$\tilde{S}(p,t)$
1.30	1.4712	0.0250	0.1843	0.1729	1.21	2.0208	0.0343	0.2490	0.2251
1.69	0.2950	0.0039	0.0376	0.0506	1.64	0.4023	0.0062	0.0527	0.0650
1.80	0.0025	0.0118	0.0009	0.0017	1.79	0.0042	0.0087	0.0030	0.0052
1.93	0.0574	0.0002	0.0007	0.0009	1.93	0.0565	0.0002	0.0007	0.0009
1.96	0.0010	0.0000	0.0002	0.0032	1.96	0.0014	0.0000	0.0001	0.0031
1.97	0.0455	0.0026	0.0025	0.0032	1.97	0.0472	0.0026	0.0036	0.0040
2.21	0.0289	0.0002	0.0000	0.0000	2.21	0.0293	0.0002	0.0000	0.0000
2.40	0.0457	0.0002	0.0028	0.0380	2.39	0.0600	0.0003	0.0046	0.0368
2.71	0.0036	0.0000	0.0095	0.0145	2.71	0.0036	0.0000	0.0095	0.0145
2.81	0.0610	0.0000	0.0072	0.0005	2.81	0.0610	0.0000	0.0072	0.0005
3.02	0.0418	0.0008	0.0065	0.0014	3.01	0.0101	0.0002	0.0022	0.0005
3.08	0.0028	0.0008	0.0043	0.0001	3.08	0.0028	0.0008	0.0043	0.0001
3.13	0.0004	0.0000	0.0006	0.0117	3.13	0.0004	0.0000	0.0006	0.0117
3.16	0.0084	0.0030	0.0000	0.0004	3.16	0.0079	0.0030	0.0000	0.0006
3.21	0.0000	0.0000	0.0083	0.0027	3.21	0.0001	0.0000	0.0088	0.0028
				$\sum_n S_n = 0.30$					$\sum_n S_n = 0.37$

of the 5., 6., 8., 13. and 14. states agree with the corresponding experimental values [4]. We observe from the experimental data of ^{158}Gd that weak $B(E2)$ values of the first state imply that the first 0^+ state might be pairing vibrational or a mixture of pairing and β-vibration mode. By studying the related experimental data of 0^+ states, Garrett concluded that any state to be properly labeled as β-vibrational should have $B(E2; K^\pi = 0^+ \to 2^+_{gsb})$ values of 12–33 w.u. and $\rho^2(E0) \times 10^3$ values of 85–230 and weak two-nucleon transfer strengths [3]. As is seen from the tables neither of the states has a sufficient quadrupole collectivity to be accepted as a β-vibrational state. In the result, the calculated values also imply a pairing vibrational mode for the first 0^+ state, they might be not a completely pure mode, though. Furthermore, theoretical $\tilde{S}_n(p,t)$ values show that only the 1.,2.,8.,9. and 13. states

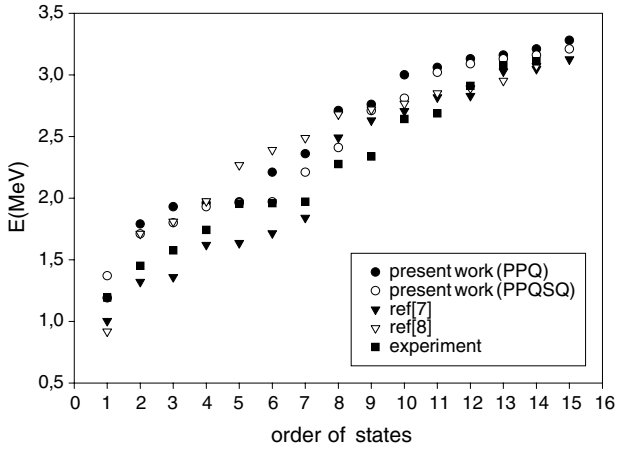

Fig. 1. Distribution of the calculated and experimental energies of ^{158}Gd.

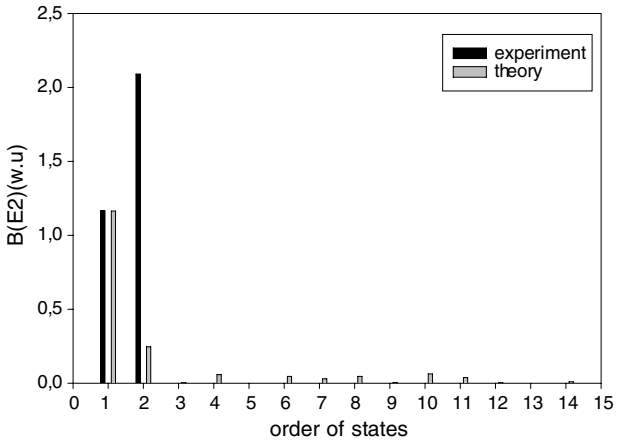

Fig. 2. Calculated and experimental $B(E2)$ values of the ^{158}Gd (data from ref. [8]).

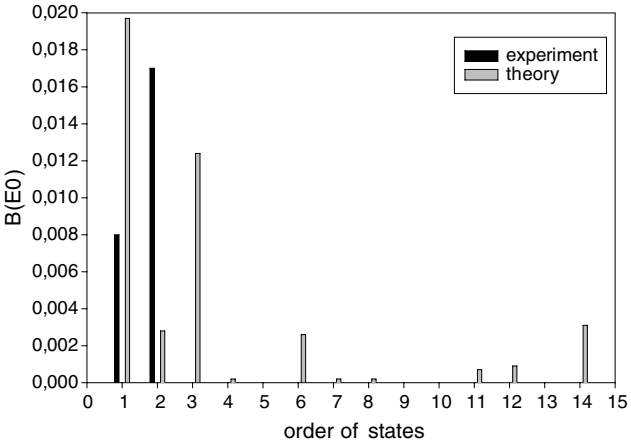

Fig. 3. Calculated and experimental $B(E0)$ values of the ^{158}Gd (data from refs. [8,21]).

can be pairing vibrational states. Naturally, the remaining states can be 2qp in character. Since, there is not a recent (t,p) experiment on the ^{158}Gd isotope, the experimental spectroscopic factors corresponding to the measured states in [20] can be used for comparison. Thus we conclude that our theoretical (t,p) spectroscopic factors agree with the data for the two lowest states.

In fig. 1, the experimental energy values have been compared with the calculated values of the present work (PPQ and PPQSQ models) and of refs. [7,8]. The results of table 1 have been used for the present work. Sun *et al.* successfully obtained a large number of 0^+ states up to ~ 3.5 MeV as a 2 or 4 quasiparticle configuration. However, although they calculated seven 0^+ below ~ 2 MeV as in the experimental case, there is a gap in their calculated energy values located between ~ 1850 keV and ~ 2500 keV, *i.e.* a gap of 650 keV [7]. On the contrary, the experiment gives 5 states in that region, *i.e.* at 1954, 1960, 1972, 2277, and 2338 keV, respectively [4]. It seems impossible to remove such a large gap and to rearrange the distribution of the states by adjusting the model parameters. Moreover, there is also a similar disagreement in the IBM results for the distribution of the states [6]. Zamfir *et al.* showed that by taking into account one f and one p boson, IBM is able to describe the number of states increasing their energies up to 4 MeV. It is clear from fig. 1 that the most consistent distribution belongs to the present model (PPQSQ).

In fig. 2, the $B(E2; K^\pi = 0^+ \rightarrow 2^+_{gsb})$ transition probabilities have been presented and compared with experiments. The theoretical $E2$ decays of the 0^+ states to the 2^+ground state are all weak except the first two ones. The present model gives the best values for the first state and also the second state. While the first state is in good agreement with the data, a relatively large $B(E2)$ value can be obtained for the second state (~ 0.25 w.u.). As stated in refs. [8,7], Iudice *et al.* could find ~ 0.03 w.u. and Sun *et al.* could find only ~ 0.004 w.u. for the $B(E2)$ value for the second state, respectively. Whereas, as is seen from the tables, it is possible to increase the $B(E2)$ values for the first two states by changing the interaction strength parameters.

In fig. 3, the $\rho^2(E0)$ transition probabilities have been presented and compared with the data. Experimentally, like the $E2$ decays a similar situation exist also for the monopole transitions, *i.e.* the first strength is weaker than the second one. However, the model gives a stronger value for the first state and a weaker value for the second one. Moreover, there exist also a few appreciable strengths for the 3., 6., and 14. states.

In fig. 4, the spectroscopic factors for (p,t) reactions have been presented and compared with the data. The mentioned problem also continues for the $\tilde{S}_n(p,t)$ values: a negligible strength is collected in the first one the strongest strength is collected in the second one. The present model gives about 0.143 and 0.043 for the spectroscopic factors of the first and the second states, respectively, exactly the opposite of the data. However, the theoretical summed strength of the (p,t) reaction for the parameters in table 1 is in agreement with the data, as is seen in the right lower corner of table 1, *i.e.* both of them are about 25% of the ground-state spectroscopic factor [4].

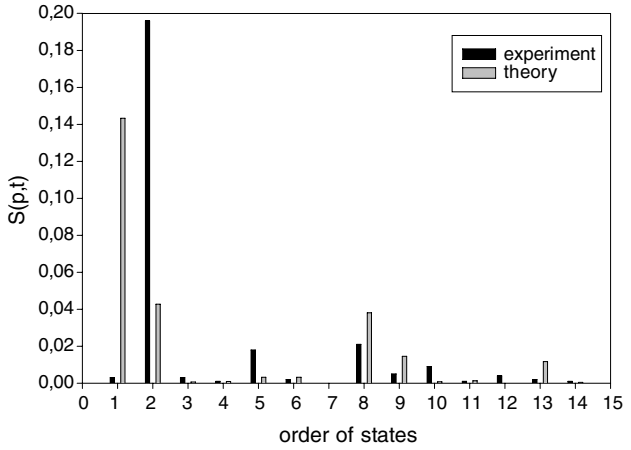

Fig. 4. Calculated and experimental spectroscopic factors for (p,t) reaction in ^{158}Gd (data from ref. [8]).

Nevertheless, obtaining a considerable $\tilde{S}_n(p,t)$ value for the second state is an important development for the theory, since, in ref. [8], Iudice *et al.* obtained only 0.006 and 0.01 for the $\tilde{S}_n(p,t)$ value of the second state, by using the 0.26 and 0.29 quadrupole deformations, respectively.

5 Conclusion

Recent observations of a large number of 0^+ states in heavy deformed regions opened a new window and emphasized the importance of the microscopic approach to the atomic nucleus. In fact studies in this field imply the need for new models and approximations based on the microscopic approach. Specifically by using the spin-quadrupole interaction here, we corroborate that this interaction produces an extra collective 0^+ state below the gap. Thus a good distribution of the states can be obtained up to ~ 3 MeV. In any case, as in earlier studies [7,8] introducing the two phonon terms to the model might limit the number of 2qp states, especially above $\sim$ 2.5 MeV. As is stated before, it is possible to find a relatively large strength for the $E2$, $E0$ decays and spectroscopic factors of the second state by means of the spin-quadrupole interaction. However, it is not possible to suppress the strengths of the mentioned quantities for the first state by means of the spin-quadrupole interaction. This is a difficult theoretical problem to overcome. In addition, it seems impossible to solve the problem using quadrupole-pairing instead of spin-quadrupole interaction [8]. Because earlier experiments show that the opposite situation exists for the (t,p) reaction [20], *i.e.* the (t,p) reaction strength of the first state is stronger than that of the second one. However, by using spin-quadrupole interaction, finding relatively large strengths for the collectivity of the second state is a significant development for the theory. In fact for a pure spin-quadrupole state, the spectroscopic factors should be negligible, since spin-quadrupole forces have no diagonal matrix elements, so wave functions for the pure spin-quadrupole states can include only the quasiparticles on different single-particle levels. Then the large calculated spectroscopic factors of the second state display strong coupling between the spin-quadrupole and other forces. Furthermore, it is clear from the secular equation (14) that the first term, which contains pairing effect, gives a contribution for the formation of all states. As a consequence, there is an indirect effect of the pairing interaction on the collectivity of the spin-quadrupole excitation. Thus, the indirect coupling in question increases the pairing collectivity *i.e.* the value of the spectroscopic factors.

The whole results show that without using the octupole-octupole interaction with two phonons, it is possible to obtain a fairly good description such a large number of 0^+ states in the framework of the present model.

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